



A systematic review of factors associated with high schoolers' algebra achievement according to HSLs:09 results

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Abstract

In this study, the authors examined a group of related literatures that used the same large-scale nationally representative dataset with algebra achievement as the outcome to explore why results from the same dataset may differ across studies. More specially, the authors synthesized the extant research literature that utilized data from the High School Longitudinal Study of 2009 (HSLs:09) to investigate the student-, teacher-, school-, and parent-level characteristics associated with algebra achievement and whether these relationships were consistent across students' backgrounds (i.e., race/ethnicity, gender, socio-economic status, and prior achievement). The authors have summarized the results of 21 studies across 2 outcome variables (ninth- and eleventh-grade algebra achievement) using optimal resource theory as a framework to summarize evidence-based practices for improving student outcomes.

Keywords Systematic review · Algebra achievement · High school · HSLs:09 data

They say it takes a village to raise a child, but does it take a village to educate one? This research explores the idea of how the “village” of parents, schools, teachers, and students influences algebra achievement, while examining any differences attributable to race/ethnicity, socio-economic status (SES), gender, or prior achievement.

According to What Works Clearinghouse (2020) “Algebra moves students beyond an emphasis on arithmetic operations to focus on the use of symbols to represent numbers and express mathematical relationships” (p. 1). This use of symbols provides the language in which students can study the patterns in mathematics, hence an understanding of algebra is essential for success in advanced mathematics courses. However, students' difficulties with algebra are well documented and for many, algebra acts as a barrier to higher-level mathematics and other STEM courses (Bednarz, 2001; Booth, 1984; Kieran, 2007; MacGregor, 1996; Sharpe, 2019). Critically, this barrier disproportionately affects Black and Latino students and students from low-income backgrounds (NAEP, 2015). Therefore, the primary focus of this research will be to explore what factors at the parent, school, teacher, and

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student levels are associated with algebra achievement while examining any differences across race/ethnicity, SES, gender, or prior achievement groups.

The advancement of knowledge and development of interventions should be built on prior existing work (Xiao & Watson, 2017), but to do this, we need to have an accurate understanding of prior work and why results may be inconsistent across studies. In this study, the authors conduct a systematic review to analyze and synthesize a group of related literatures. This systematic review differs from other published reviews because we examined a group of related literatures that used the same large-scale nationally representative dataset with algebra achievement as the outcome to explore why results from the same dataset may differ across studies.

1 Review of literature

1.1 Algebra as a core subject

It is widely recognized that mathematics achievement is essential for success in Science, Technology, Engineering, and Mathematics (STEM) based fields. Despite this, many students lack the mathematical (especially algebraic) skills needed to qualify for jobs in a competitive global market. Proficiency in algebra is a gatekeeper to success in advanced mathematics courses, high school graduation, college admission and completion, and obtaining employment in STEM careers (Hughes et al., 2014; National Mathematics Advisory Panel, 2008; Vogel, 2008).

Algebra continues to play an important role in the precollege curriculum with national policy documents, such as the NCTM Principles and Standards for School Mathematics (2000) and NCTM's document Catalyzing Change in High School Mathematics: Initiating Critical Conversations (2018), continuing to emphasize how algebra serves as a gateway course to success in future STEM courses and careers. With the lack of proficiency among high school students (NAEP, 2015), algebra may be acting as a gatekeeper instead of a gateway course to higher-level STEM courses; this may be a result of how algebra is taught by teachers or offered by schools. This differential access to algebra has disproportionately excluded Black, Latino, and low-SES White students from college preparatory mathematics classes (Moses & Cobb, 2001). Prior research has indicated there may be more effective methods to teach students algebra at the secondary level, for example, a function-based approach to teaching algebra may be more effective in introducing students to algebra than the more traditional methods that focus on the manipulation of variables (Sharpe & Schliemann, 2017; Stroup et al., 2005). A function-based approach to algebra instruction takes the fundamental concepts of variables (i.e., describing real world quantities whose values change), functions (i.e., describing the relationships among those quantities), and the multiple representations of functions as central to teaching algebra (Heid, 1996).

Researchers have found that taking algebra in eighth grade was associated with higher mathematics achievement when the students were prepared to take algebra earlier (Domina, 2014; Rickles, 2013). In addition, Filer and Chang (2008) showed that peer and parent influences were directly related to students' taking algebra earlier, while taking Algebra II in high school resulted in greater college readiness (Gaertner et al., 2014; Kim et al., 2015). Furthermore, when parents discussed college plans with their children, the children had a greater chance of attending college (Sciarra, 2010).

1.2 Students' difficulties with algebra

There have been well-documented difficulties with algebra among middle and high school students (Bednarz, 2001; Booth, 1984; Kieran, 2007; MacGregor, 1996). Algebra difficulties are noteworthy in middle school as students are asked to transition from arithmetic to algebra (Sharpe, 2019) and use letters to represent sets of indeterminate quantities (Ely & Adams, 2012). In high school, algebra difficulties tend to arise when students are asked to generate equations from real world scenarios and use the equations to solve a problem (Kieran, 2007). Students also have noted difficulties operating on unknowns and fail to understand that equivalent transformations on both sides of an equation do not alter its truth value (Bednarz, 2001; Bednarz & Janvier, 1996; Filloy & Rojano, 1989; Kieran, 1985, 1989; Steinberg et al., 1990).

According to Gueudet and colleagues (2016), “conceptual change researchers agree that some things are difficult to learn and thereby take time” (p. 10). The learning of algebra is difficult for some students and should be expected to take time. From the student perspective, the nature of mathematics has changed from simply knowing arithmetic operations, to a complex understanding of symbols that represent both quantities and mathematical relationships (What Works Clearinghouse, 2020); this is a whole new mathematical language (Yerushalmy, 2005) and students experience difficulties making this critical transition. However, from the teacher perspective, the nature of mathematics changes as the teacher transitions between learning university mathematics to teaching school mathematics. According to Ball and colleagues (Ball et al., 2005), this is the difference between content knowledge (knowledge of the content, i.e., algebra) and pedagogical content knowledge (knowledge of the content and how to teach the content). Even though these two knowledges are highly correlated and hard to separate (Blömeke et al., 2014), teachers require adequate and in-depth learning opportunities to develop both content knowledge and pedagogical content knowledge (Gueudet et al., 2016) in algebra to help make the transition from arithmetic to algebra smoother for students. For example, meanings for symbols may be obvious to teachers but not to students; teachers need to be cognizant of this and guide students to pay close attention to symbols and their referents (Harel et al., 2008).

Researchers have shown that immediate remediation, that is students enrolling in their regular algebra course and simultaneously being provided with remediation (tutorials, double-dose, etc.), can prepare students for subsequent course content (Forrest et al., 2017; Martinez et al., 2016; Nomi & Allensworth, 2009). In addition, both teacher support (i.e., establishing caring relationships, learning about students' needs and strengths, and providing encouragement) and instructional practices (i.e., standards-based mathematics instruction or emphasizing procedural skills) have been acknowledged as playing important roles in students' academic success (Hiebert & Grouws, 2007; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010; Roorda et al., 2011).

1.3 Algebra and equity

The National Council of Teachers of Mathematics posits that all students should have an equal opportunity to learn high-quality mathematics (NCTM, 2000); however, the opportunity to learn mathematics has frequently been correlated with SES (Schmidt & McKnight, 2012; Schmidt et al., 2015). Inequalities across socioeconomic strata are an issue for many

countries (Organisation for Economic Co-operation and Development (OECD), 2020), and are associated with students' unequal opportunities to learn crucial mathematics. Schmidt and colleagues (2015) analyzed data from the 2012 Programme for International Student Assessment (PISA) exploring the relationship between opportunity to learn, SES, and mathematics literacy. They concluded that, among the countries in the PISA data, the link between higher SES and higher mathematical literacy is explained in part by greater exposure and opportunities to learn mathematics.

The USA has experienced similar patterns among SES, opportunity to learn, and mathematics achievement, but critically, these factors are also correlated with race and ethnicity. Socioeconomic disparities have limited Black and Latino students' access to high-quality algebra instruction and disproportionately excluded them from opportunities to pursue college preparatory mathematics (Moses & Cobb, 2001). Also tied with SES and contributing to inequities among Black and Latino students are differences in home and school resources, including less available guidance on mathematics course-taking (Perna & Titus, 2005) and disproportional placement into lower-level mathematics course-taking trajectories, even after adjusting for prior academic performance (Irizarry, 2021).

According to the National Assessment of Educational Progress (NAEP), students across SES, gender, and racial lines performed differently on the algebra domain. NAEP evaluates a randomized sample of schools and students in Grade 12 from across the USA. The 2015 NAEP report indicates, across race/ethnicity, that White and Asian students outperformed their Black, Latino, and Indigenous counterparts; the percentage of students at or above proficiency was the highest among Asian students at 47%, and was the lowest among Black students at 7%. Across genders, the percentage of students at or above proficiency was 26% for males and 23% for females. The NAEP study also showed that the percentage of students at or above proficiency in twelfth grade algebra increases as SES increases (NAEP, 2015).

Given the important role of algebra as a gatekeeper to higher-level mathematics and the pursuit of a career in STEM fields (Hughes et al., 2014) coupled with the difficulties students experience in algebra, it is imperative that we have a clear understanding of what malleable (i.e., can be influenced directly) factors at the student, teacher, school, and parent level will lead to greater achievement (and eventually understanding) in algebra for high school students regardless of race/ethnicity, SES, gender, or prior achievement.

2 Methodological framework

This study employs optimal resource theory (ORT), to analyze and synthesize evidence-based practices for improving student achievement (Anderson, 2015). ORT is a five-principle framework that targets malleable factors within school control (e.g., teaching practices, course offerings) while also considering non-malleable factors beyond school-based control that affect student achievement (e.g., race/ethnicity, gender, SES). The five principles are as follows:

- (1) account for the multiplicity and complexity of factors that influence student development,
- (2) account for externally controlled factors where practicable,
- (3) examine manipulable factors that may be internally controlled,

- (4) focus on incremental progress or reasonable outcomes rather than on comprehensive change, and
- (5) focus on maximizing progress with available resources, despite external circumstances.

The current study focuses on the first three of the five principles that govern ORT, eventually producing a summary of findings that may be used to inform future interventions appropriate for application to the last two principles of ORT.

Our focus was on the malleable factors within the stakeholder control (i.e., student, teacher, school, or parent), on high schoolers' algebra achievement, and how these may be affected by factors beyond their control (i.e., race/ethnicity, SES, gender, and prior achievement). Our research questions were as follows:

- 1) What malleable factors at the student, teacher, school, and parent level have been found to be significantly associated with the US high school students' algebra achievement:
 - a. as reported by one study?
 - b. as reported by 2 or more studies?
- 2) Which student-, teacher-, school-, and parent-level malleable factors' association with algebra achievement vary across studies/models?

3 Method

The purpose of this study was to synthesize the research literature that utilized data from one nationally representative dataset by exploring the malleable student-, teacher-, school-, and parent-level factors associated with algebra achievement beyond differences across race/ethnicity, SES, gender, or prior achievement. A key part of the synthesis involved determining whether these relationships were consistent across studies and student backgrounds.

To answer the research questions, we performed a systematic review of the literature that used the High School Longitudinal Study of 2009 (HSLs:09) data. The HSLs:09 data were collected within the USA and are meant to be representative of 2009 ninth graders. Hence, the results of this study pertain to the USA; however, the research methodology can be applicable to other countries' nationally representative datasets.

3.1 High School Longitudinal Study of 2009

HSLs:09 is an ongoing survey that began in the fall of 2009 with approximately 20,000 ninth graders (typically age 14–15) attending 944 schools across the USA. The first follow-up was conducted in the spring of 2012 when most students were in the eleventh grade (age 16–17). Students, their parents, mathematics and science teachers, administrators, and counselors completed surveys. In addition to the survey data, students also took a cognitive mathematical assessment in the fall of 2009 and spring of 2012.

3.1.1 HSLs:09 conceptual model

The HSLs:09 dataset used a conceptual model to guide its instrument design. In this model, the student is the main unit of analysis; however, the study incorporates the

students' interactions with their families, teachers, peers, and community. The model attempts to locate the factors (e.g., student background and previous experiences, social context and interpersonal influences) that influence a students' values and expectancies which then influences their academic goal setting and decision making (Ingels et al., 2011). Students' values and expectancies can be described using the expectancy-value model (Eccles, 2009; Wigfield & Eccles, 1992) which states that students' academic goal-setting and decision-making are determined by their expectancy for success and the subjective value they attach to the tasks involved in those academic goals and decisions.

The studies included in this systematic review focus on factors that are associated with ninth- and eleventh-grade algebra achievement. The algebra assessment for the HSLS:09 was developed as a measure of algebraic preparation for the study of science, for further study within the mathematical sciences, and for the workplace.

3.1.2 Ninth-grade algebra assessment

The base-year algebra assessment for the HSLS:09 was administered to ninth graders in the fall of 2009 and was designed to measure algebra proficiency. Consisting of six algebraic content domains (algebraic language; proportions; equations, inequalities, and functions [linear and nonlinear]; systems of equations; and sequences) and four algebraic processes (demonstrating skills; using representations; reasoning; and problem solving), the test was designed to assess a balanced cross-section of algebra understanding. The assessment was field-tested with each item reviewed by an advisory panel consisting of retired or experienced mathematics professors, teachers, and education consultants. Students were administered the assessment via computer and scored on 72 items following an item response theory (IRT) design. With IRT, a student's score is an estimate of their true proficiency based on item difficulty and patterns of correct, incorrect, and omitted responses (Ingels et al., 2011, 2014).

3.1.3 Eleventh-grade algebra assessment

The first follow-up algebra assessment was administered in the spring of 2012, when most of the base-year cohort were in the eleventh grade. The initial follow-up assessment was designed to measure algebra proficiency and the change in proficiency from early to late high school. Assessment items pertained to the same algebraic content domains and processes as the base-year instrument; however, item difficulty was increased to avoid ceiling effects (Ingels et al., 2011, 2014).

In addition to the original research agenda of HSLS:09, many researchers have used these nationally representative data to explore factors associated with the US high schoolers' algebra achievement while examining differences due to race/ethnicity, SES, gender, or prior achievement. Given the importance of improving algebra proficiency in the USA, we were interested in what factors might be associated with higher achievement on this assessment.

3.2 Data sources

We performed a search of the literature through June 2017 that utilized data from HSLS:09 related to mathematics. All efforts were made to alleviate publication

bias by including published and unpublished studies in the analysis. We excluded all reports and executive summaries from our search and instead focused on journals, dissertations, research briefs, conference proceedings, and books. Data collection occurred in 4 stages: (1) identification: identify records through database searches using variations of the search terms “HSLs:09” and “mathematics”; (2) screening: screen the records’ abstracts for HSLs:09 and mathematics; (3) eligibility: screen the methods section for algebra achievement as an outcome variable; and (4) inclusion: determine the number of studies included in the qualitative synthesis. We searched the following databases: Google Scholar, ERIC, PsycINFO, Education Source, Academic Search Complete, Mental Measurements Yearbook with Tests in Print, JSTOR, Project MUSE, Sociological Abstract, and ProQuest Dissertation and Thesis A&I. We used *RefWorks* to eliminate duplicate records.

The first stage of data collection was to identify the possible records for the systematic review. Table 1 shows the search terms, the databases that were searched, the number of results given, and the number of records selected. The number of records remaining after duplicates were removed was 73.

The second stage of data collection was to screen all 73 selected abstracts for the use of the dataset HSLs:09 and connections to mathematics. From this initial screening, 3 of the records did not relate to our interests of HSLs:09 and mathematics, which left 70 remaining records. Out of the 70 records, we identified 29 articles, one working paper, 34 dissertations, 5 conference proceedings, and a singular book chapter (see Table 2).

The third stage of data collection was to screen the methods and results section of each study for the use of either ninth- or eleventh-grade algebra achievement as the outcome or dependent variable. The fourth and final stage of data collection was selecting the records for inclusion in the study based on having at least one malleable factor and no significant methodological concerns. The records with only non-malleable factors were not included. After this screening, we were left with 21 studies to review (8 journal articles, 12 dissertations, and one book chapter), see Table 2. We addressed the issue of the comparability of the included studies by including studies relevant to our research purpose and shared high-quality methodology (i.e., no significant methodological concerns). We evaluated the quality of each included study’s methodology by appraising whether the sample was clearly described, whether a clear definition was provided for the variables including which HSLs:09 items formed the variables, and whether an appropriate statistical approach was used (i.e., the complexity of the HSLs:09 dataset was taken into consideration). Given this appraisal of each study’s methodology, all included studies were given equal weighting in our analysis.

Data (descriptions of the study, variables, and methodology) were extracted from each study that met our inclusion criteria. The studies were categorized based on the two primary outcome variables: ninth-grade algebra achievement and eleventh-grade algebra achievement. Across the studies, we classified the level of the factors/predictors/independent variables as either parent-, school-, teacher-, or student-level variables, and as either malleable or non-malleable. Since socioeconomic status (SES) was defined as a composite of parent/guardian education, occupation, and family income status, we classified SES as a parent-level variable. For more detailed descriptions of the included sources, see the [supplementary materials](#).

Table 1 Identification (search for records pertaining to HSLs:09 and math)

Searched terms	Database searched	# of results	# of records selected
HSLs:09 mathematics	Google Scholar	240	70
(HSLs:09 OR “high school longitudinal study”) AND (math OR mathematics)	ERIC, PsycINFO, Education Source, Academic Search Complete, Mental Measurements Yearbook with Tests in Print	40	27
(HSLs:09 OR “high school longitudinal study”) AND (math OR mathematics)	JSTOR	9	1
HSLs:09 mathematics	Project MUSE	3	3
HSLs:09 math	Project MUSE	3	3
“High school longitudinal study” math	Project MUSE	6	3
“High school longitudinal study” mathematics	Project MUSE	6	3
(HSLs:09 OR “high school longitudinal study”) AND (math OR mathematics)	Sociological Abstracts	5	3
(HSLs:09 OR “high school longitudinal study”) AND (math OR mathematics OR algebra)	Dissertations and theses A and I	141	26

Table 2 Results from the screening, eligibility, and inclusion stages of data collection

Records	Screening	Eligibility	Inclusion
Articles	29	12	8
Book chapter	1	1	1
Conference proceedings	5	1	0
Dissertations	34	20	12
Working papers	1	0	0

4 Results

The results of the synthesis included both non-malleable and malleable variables at the student, teacher, parent, and school levels associated with high school students' algebra achievement. The non-malleable factors identified included SES, region/location of school, race/ethnicity, gender, prior achievement, language/first language, school type (public/private), over capacity enrollment, immigrant generation (1st/2nd), parent education, and teaching experience (years). Our primary concern however was the identification of the malleable independent variables, since these can be influenced directly. We reported all significant ($p < 0.05$) malleable independent variables that were shown to be significant in at least one study. The studies did not provide sufficient information to compute and compare effect sizes. Several malleable factors had varying associations with algebra achievement across studies/models, and for these instances, we have provided a more detailed analysis. Given that not all the factors within a study (or the HSLS:09 dataset) were used by multiple studies, we reported the statistically significant factors reported by one study, to get a better idea of all the factors that were associated with higher algebra achievement for ninth- and eleventh-grade students in the USA. To answer research question 1a, Tables 3, 4, and 5 provide the results for the student, school, and teacher-level malleable factors that were found to be significantly associated with the US high schoolers' algebra achievement, as reported by one study. Note that there were no factors at the parent level.

The significant student level- factors reported by a single study are shown in Table 3. There were eight positive predictors and three negative predictors of ninth-grade algebra achievement. In terms of eleventh-grade algebra achievement, there were five positive predictors and two negative predictors. The following factors were defined as follows: academic resilience is "individuals' positive educational achievement outcomes despite exposure to statistically significant risk factors" (He, 2014, p. 7); sense of school belonging refers to "the degree to which a student feels accepted, respected, and included by both fellow students as well as adults within the school environment" (Lar-rain, 2015, p. 6); mathematics self-efficacy refers to a student's confidence in their ability to do well in their math class; mathematics interest refers to the degree to which a student finds their math class enjoyable; mathematics identity refers to the degree to which a student sees themselves to be a "math person" and perceives others to see them as a "math person"; mathematics utility is the degree to which a student finds math useful for college, career, and adult life (Ingles et al., 2011); and mathematics self-concept is a composite of mathematics self-efficacy, identity, utility, and ability mindset (i.e., whether a student believes most people can learn to be good at math and/or they have to be born good at math; Hale, 2017). For more detailed descriptions of the variables, see the [supplementary materials](#).

Table 3 Student-level factors significantly associated with algebra achievement reported by one study

Factor	Effect	Sample	Source(s)
On 9th-grade algebra achievement			
Academic resilience	Positive	Low achievers	He (2014)
At or above pre-algebra in 8th grade	Positive	Enrolled and select math courses in fall 2009 with no missing data	Carolan and Matthews (2015)
At or above Algebra I in 9th grade	Positive	Enrolled and select math courses in fall 2009 with no missing data	Carolan and Matthews (2015)
Highest level of education expected to attain	Positive	All students	Kim-Choi (2015)
Hours spent on math outside class	Negative	All students	Larain (2015)
Consulted a variety of people about math coursework	Positive	Enrolled and select math courses in fall 2009 with no missing data	Carolan and Matthews (2015)
Enrolled in remedial math in 9th grade	Negative	All students	Larain (2015)
Felt math teacher supported all students' learning	Positive	African American Males	Onsongo (2015)
Math self-efficacy $\times$ sense of school belonging	Positive	All students	Larain (2015)
Math self-efficacy $\times$ African American student	Negative	All students	Larain (2015)
Math interest $\times$ consulted a variety of people about math coursework	Positive	All students	Carolan and Matthews (2015)
On 11th-grade algebra achievement			
College plans	Positive	All students	Hale (2017)
Importance of family in college plans	Positive	Latina/o Americans	Fajardo, Hernandez, and Muñoz (2016)
Math course-taking plans	Positive	All students	Hale (2017)
Took Algebra I in 9th grade	Positive	African Americans	Alexander (2015)
Math self-concept	Negative	All students	Hale (2017)
Math self-efficacy $\times$ African American	Negative	All students	Larain (2015)
Math self-efficacy $\times$ Pacific Islander	Positive	High performing 9th grade students	Kotok (2017)

Table 4 School-level factors significantly associated with algebra achievement reported by one study

Factor	Effect	Sample	Source(s)
On 9th-grade algebra achievement			
Collective educational goal $\times$ parent SES	Negative	All students	Min and Goff (2016)
On 11th-grade algebra achievement			
Required 4 years of math and 3 years of science for graduation	Positive	All students	Saw (2016)
Offered AP courses	Positive	All students	Saw (2016)
Pressed student academics	Positive	All students	Saw (2016)

At the time of the HSLS:09 study, students in the USA were typically exposed to pre-algebra in Grades 6 to 8, Algebra I in Grade 9, and Algebra II in Grade 11. Pre-algebra typically allows students to apply and extend previous understandings of arithmetic to algebraic expressions; Algebra I covers expressions, equations, graphs, lines, exponents, radicals, polynomials, and introduction to functions; Algebra II covers functions and polynomials, complex numbers, exponential and logarithmic functions, graphs and transformations, sequences and series, and introduction to trigonometry (CME Project Development Team, 2009a, 2009b; NGA Center & CCSSO, 2010).

Factors at the school level found to be significant and reported by a single study are shown in Table 4. There was one negative predictor of ninth-grade algebra achievement and three positive predictors of eleventh-grade algebra achievement. Collective educational goals referred to teachers at this school feel responsible that all students learn. Offered AP courses referred to whether the school offered advanced placement courses. Pressed student academics referred to teacher, counselor, and principal academic expectations for students as perceived by counselor.

Table 5 Math teacher-level factors significantly associated with algebra achievement reported by one study

Factor	Effect	Sample	Source(s)
On 9th-grade algebra achievement			
Emphasized procedures	Negative	All students with no missing data	Yu and Singh (2016)
Emphasized concepts	Positive	All students with no missing data	Yu and Singh (2016)
Teacher believed students can overcome adversity $\times$ African American student	Negative	Students enrolled in math courses in 9th grade	Rochmes (2014)
Teacher believed students can overcome adversity $\times$ African American student $\times$ parent SES	Positive	Students enrolled in math courses in 9th grade	Rochmes (2014)
Teacher believed students can overcome adversity $\times$ multiracial student $\times$ parent SES	Positive	Students enrolled in math courses in 9th grade	Rochmes (2014)
On 11th-grade algebra achievement			
Emphasized concepts	Positive	Hispanic Americans	Maldonado (2016)
Perception of overall class achievement (average, above average, highly variable)	Positive	All students	John (2017)

Table 6 Student-level factors significantly associated with algebra achievement reported by 2 or more studies

Factor	Effect	Sample	Source(s)
On 11th-grade algebra achievement			
Highest level math course taken by 11th grade	Positive	All students	Froiland and Davison (2016)
		Native Hawaiian or Pacific Islanders	Froiland et al. (2016)
On advanced math track	Positive	African Americans	Alexander (2015)
		All students	John (2017)
		High performing students	Kotok (2017)

Table 7 Parent-level factors significantly associated with algebra achievement reported by 2 or more studies

Factor	Effect	Sample	Source(s)
On 11th-grade algebra achievement			
Highest level of education parent expected student to attain	Positive	All students	Carolan (2016), Froiland and Davison (2016)

Factors at the mathematics teacher level that were found to be significant and which were reported by a single study are shown in Table 5. There were three positive predictors and two negative predictors of ninth-grade algebra achievement. There were two positive predictors of eleventh-grade algebra achievement. Emphasized procedures referred to math teacher's reported emphasis on algorithms, procedures, speedy and accurate computations, and standardized test preparation. Emphasized concepts referred to math teacher's reported emphasis on concepts, student interest, problem solving, reasoning, connecting ideas, preparing for future study, explaining, and the nature of math.

To answer research question 1b, Tables 6 and 7 provide the results for the student- and parent-level malleable factors that have been found to be significantly associated with the US high schoolers' algebra achievement, as reported by two or more studies. Note that there were no factors at the school or teacher level.

Two student-level factors were found to be significantly positive predictors of eleventh-grade algebra achievement as evidenced by two or more studies (see Table 6).

There was one parent-level factor found to be a significant positive predictor of eleventh-grade algebra achievement reported by two or more studies (see Table 7).

To answer research question 2, we described the student-, school-, and parent-level malleable factors that had varied effects across studies and/or models. Note that there were no variations across studies/models in our review at the teacher level.

4.1 Student level

There were seven factors that had varied effects across the studies at the student level: peer engagement, school engagement, math effort, math identity, math

Table 8 Student-level factors associated with algebra achievement that vary across studies/models

Factor	Grade of algebra achievement outcome	Effect	Sample	Source(s)
Peer engagement	11th	+	• African Americans • Hispanic/Latin Americans	Alexander (2015); Fajardo, Hernandez, and Munoz (2016); Kotok (2017)
Peer engagement	11th	0	• All	Carolan (2016)
School engagement	9th and 11th	+	• All • Low achievers	John (2017); Kim-Choi (2015)
School engagement	9th and 11th	0	• African Americans • High achievers	Alexander (2015); Kim-Choi (2015)
Math effort 9	9th	-		Middleton (2013)
Math effort 11	11th	+		John (2017)
Math identity 9	9th and 11th	+	• All • High achievers • African Americans	Alexander (2015); Briggs (2014); Carolan (2016); Kim-Choi (2015); Kotok (2017)
Math identity 11	11th	+	• All females	Fajardo, Hernandez, and Munoz (2016); Howard (2015); John (2017)
Math interest 9	9th	-	• All	Middleton (2013)
Math interest 9	9th	0	• All • High achievers	Carolan and Matthews (2015); Kim-Choi (2015)
Math interest 9	9th and 11th	+	• Low achievers • Native Hawaiian/Pacific Islanders	Kim-Choi (2015); Froiland and Davison (2016); Froiland et al. (2016)
Math interest 11	11th	-	• All females • White males	Howard (2015)
Math interest 11	11th	0	• African American females • Asian American females • Hispanic/Latin American females	Howard (2015); John (2017)
Math self-efficacy 9	9th	+	• All • High-achievers • African American males	Briggs (2014); Kim-Choi (2015); Kotok (2017); Larrain (2015); Onsongo (2015); Yu and Singh (2016)
Math self-efficacy 9	9th and 11th	0	• African Americans • Low-achievers	Alexander (2015); Kim-Choi (2015)
Math self-efficacy 11	11th	+	• All females • White females	Howard (2015); John (2017)
Math self-efficacy 11	11th	0	• Hispanic/Latin Americans • African American females • Hispanic/Latin American females • Asian American females	Fajardo et al. (2016); Howard (2015)
Math utility 9	9th	-	• All • High-achievers	Kim-Choi (2015)
Math utility 9	9th	0	• Low-achievers	Kim-Choi (2015)
Math utility 11	11th	0	• Hispanic/Latin Americans	Fajardo et al. (2016); John, (2017)

“+” means significant positive effect, “-” means significant negative effect, and “0” means an insignificant effect

Table 9 School-level factors associated with algebra achievement that vary across studies/models

Factor	Grade of algebra achievement outcome	Effect	Sample	Source(s)
Collective education goals of a school	9th	+	Accounted for SES and language variations	Min and Goff (2016)
Collective education goals of a school	9th	0	Accounted for race/ethnicity variations	Min and Goff (2016)
Percentage of advanced placement (AP) enrollment	9th	+	All students	Larrain (2015)
Percentage of advanced placement (AP) enrollment	11th	0	All students	Saw (2016)

“+” means significant positive effect, “–” means significant negative effect, and “0” means an insignificant effect

interest, math self-efficacy, and math utility (see Table 8). Peer engagement referred to whether the student’s best friend was interested in school, planned to attend college, talked about courses to take, and planned on attending class regularly. School engagement referred to arriving on time and prepared for class. Math effort referred to how often the student paid attention in math class, turned in assignments on time, and kept trying in class.

4.2 School level

There were two factors that had varied effects across the studies and/or models at the school level: collective education goals of a school and percentage of advanced placement (AP) enrollment (see Table 9). Collective education goals of a school referred to whether teachers at the school feel responsible and make sure that all students learn. Percentage of advanced placement (AP) enrollment is the percentage of the total student body enrolled in advanced placement courses.

4.3 Parent level

Parent involvement (participation as reported by parents in in-school and out-of-school activities and advice on coursework, college, and careers) was the only factor that had varied effects across the studies and/or models at the parent level (see Table 10).

Table 10 Parent-level factors associated with algebra achievement that vary across studies/models

Factor	Grade of algebra achievement outcome	Effect	Sample	Source(s)
Parent involvement	11th	+	• All females • African American females • White females	Carolan (2016); Howard (2015)
Parent involvement	11th	0	• Hispanic/Latin American females • Asian American females	Howard (2015)

“+” means significant positive effect, “–” means significant negative effect, and “0” means an insignificant effect

5 Discussion

Algebra achievement has been considered the gatekeeper to higher-level mathematics and STEM careers (National Mathematics Advisory Panel, 2008). With this in mind, it is imperative to understand how different malleable variables at the student, teacher, parent, and school levels may be related to algebra achievement beyond students' race/ethnicity, SES, gender, and prior achievement. The nature of the surveys in the HSLs:09 data are self-reports; the factors at the student level were reported by students, factors at the teacher level were reported by mathematics teachers, factors at the school level were reported by administrators and/or counselors, and factors at the parent level were reported by parents of the students.

5.1 Algebra as a core concept

Acknowledging the importance of algebra as a gatekeeper to these higher-level mathematics and STEM majors/jobs, we explored what factors impacted students' algebra achievement as it relates to algebra as a core concept. What we found was that regardless of the level of the factor (i.e., student, school, or parent), the factors explored in the studies reviewed were all positively associated with algebra achievement. The students who reported that they take pre-algebra by eighth grade, Algebra I by ninth grade, are on the advanced math track, or plan to take the highest-level math course offered by eleventh grade, were more likely to demonstrate higher algebra achievement in ninth and eleventh grades (Alexander, 2015; Carolan & Matthews, 2015; Froiland & Davison, 2016; Froiland et al., 2016; John, 2017; Kotok, 2017). Similarly, students who reported making plans for math course-taking and consulting a variety of people about those plans, making plans for college, considering the importance of family in those college plans, and holding high expectations for their own educational attainment, tended to have higher ninth- and eleventh-grade algebra achievement (Carolan & Matthews, 2015; Fajardo et al., 2016; Hale, 2017; Kim-Choi, 2015). These results were also consistent for high-performing and African American students who reported being on an advanced track in mathematics (Alexander, 2015; John, 2017; Kotok, 2017); African Americans who reported taking Algebra I by ninth grade (Alexander, 2015), Native Hawaiian/Pacific Islander students who reported taking higher-level math courses by eleventh-grade (Froiland & Davison, 2016; Froiland et al., 2016), and Latina/o American students who considered the importance of family when making college plans (Fajardo et al., 2016) all had positive effects on their ninth- or eleventh-grade algebra achievement.

Attending schools that offered AP courses was associated with higher eleventh-grade algebra achievement (Saw, 2016). Similarly, higher percentage of school-wide AP enrollment was associated with higher ninth-grade algebra achievement (Larrain, 2015). However, when Saw (2016) controlled for whether schools offered AP courses, percentage of AP enrollment was not related to students' eleventh-grade algebra achievement (Saw, 2016). These findings may suggest that the observed association between school-wide AP enrollment and algebra achievement is more a sign of school resources (e.g., available and qualified teachers for AP courses) than of the impact of AP courses themselves. In addition, students who attended schools that require 4 years of math for graduation, 3 years of science for graduation, and pressed student academics tended to have higher eleventh-grade algebra achievement (Saw, 2016). Parental report of a high level of educational attainment

expected from their child was linked to higher eleventh-grade algebra achievement (Carolan, 2016; Froiland & Davison, 2016).

5.2 Students' difficulties with algebra

In this study, we explored what specific factors positively or negatively influence students' difficulties with algebra. According to our results, the students who reported enrolling in remedial math classes in ninth grade and spending more hours on math outside of class were associated with lower ninth-grade algebra achievement (Larrain, 2015). One explanation for this result could be that since the students were enrolled in remedial courses then they were initially struggling with mathematics before spending more time on math outside of class, so their lower achievement is despite the additional efforts. An alternative to remediation which research has shown to be effective is taking support courses simultaneously with regular courses (double-dose) which can also better prepare students for subsequent course content (Forrest et al., 2017; Martinez et al., 2016; Nomi & Allensworth, 2009). Low achieving students with academic resilience and African American males who felt that their math teacher supported all students learning were more likely to have higher ninth-grade algebra achievement (He, 2014; Onsongo, 2015).

Teachers who reported that they emphasized procedures were associated with lower ninth-grade algebra achievement (Yu & Singh, 2016), while teachers who emphasized concepts were associated with higher ninth- and eleventh-grade algebra achievement (Maldonado, 2016; Yu & Singh, 2016). Given the correlational nature of the studies, it is unclear whether these results mean an emphasis on procedures caused lower achievement or whether teachers emphasized procedures more because they taught lower achievers. Similarly, it is unclear if teaching concepts led to higher achievement or teachers who taught high achievers tended to emphasize concepts. In addition, whether teachers' perceptions of their overall class achievement were average, above average, or highly variable was associated with higher eleventh-grade algebra achievement (John, 2017).

5.3 Algebra and equity

Based on the ORT framework (principles 1 to 3), the studies reviewed accounted for several factors that may influence algebra achievement. Many of these factors fall within a school's realm of influence, although we see that ORT recognizes these malleable factors likely operate differently depending on external circumstances. After examining why the same factors had varied effects across studies/models, we found that the apparent differences could be explained by external factors (e.g., different sub-samples based on race/ethnicity, gender, socioeconomic status, or prior achievement). Students experience an inequitable access to algebra based on non-malleable background factors, which often result in achievement gaps across race/ethnicities, gender, and SES (NAEP, 2015). Factors at all levels of the "village" impact disparate algebra performance among diverse students.

Peer engagement, that is having a best friend who was interested in school, made no significant difference in eleventh-grade algebra achievement for all students (Carolan, 2016), but when focusing on African American or Latin American students who reported having academically engaged peers these students tended to have higher eleventh-grade algebra achievement (Alexander, 2015; Fajardo et al., 2016; Kotok, 2017).

Although higher levels of school engagement (when a student arrives on time and brings proper materials for class) were found to be associated with higher ninth- and eleventh-grade algebra achievement for all students including low-achieving students in some studies (John, 2017; Kim-Choi, 2015), other researchers found no significant relationship between school engagement and ninth or eleventh- grade algebra achievement for African American students or high-achieving students (Alexander, 2015; Kim-Choi, 2015).

Students in ninth-grade mathematics who often paid attention in class, turned in assignments, and kept trying in class (math effort), tended to have higher eleventh-grade algebra achievement (John, 2017), while students who put in little math effort tended to have lower scores on ninth-grade algebra achievement (Middleton, 2013). In Grade 9, if a student and others perceived the student as a “math person” (math identity), then all students (especially, high-achieving students, or African American students) demonstrated higher ninth- and eleventh-grade algebra achievement (Alexander, 2015; Briggs, 2014; Carolan, 2016; Kim-Choi, 2015; Kotok, 2017). Additionally, female students with a strong math identity in Grade 11 tended to have higher eleventh-grade algebra achievement (Fajardo et al., 2016; Howard, 2015; John, 2017).

In an unexpected result from a single study, ninth-grade students who were interested in math (that is, they reported it was their favorite course and they enjoyed it) were more likely to show lower algebra achievement in ninth grade (Middleton, 2013). Several other researchers found no relationship between ninth-grade students' interest in math and ninth-grade algebra achievement, especially for high-achieving students (Carolan & Matthew, 2015; Kim-Choi, 2015). Kim-Choi (2015) found that low-achieving ninth-grade students who were interested in math tended to have higher ninth-grade algebra achievement. Ninth-grade students who were interested in math were also more likely to have higher eleventh-grade algebra achievement, especially Native Hawaiian/Pacific Islanders (Froiland & Davison, 2016; Froiland et al., 2016). Results regarding the variable of ninth-grade math interest were inconclusive in Middleton's (2013) study, wherein math interest was a significant negative factor, and Kim-Choi (2015) wherein it was an insignificant factor for all students. In taking a deeper look at the survey items for math interest, we found that Middleton only used a portion of the items used by Kim-Choi, which accounts for the differences in significance.

Interest in math measured in eleventh grade was found to be negatively associated with eleventh-grade algebra achievement for all females and White females (Howard, 2015), while no relationship was found between eleventh-grade interest in math and eleventh-grade algebra achievement for African American females, Asian American females, or Latin American females (Howard, 2015; John, 2017).

Students' confidence in their ability to do math (self-efficacy) as measured in ninth grade was related to higher ninth-grade algebra achievement, especially among high-achieving students and African American males (Briggs, 2014; Kim-Choi, 2015; Kotok, 2017; Larrain, 2015; Onsongo, 2015; Yu & Singh, 2016). There was no relationship found between ninth-grade students who were confident in their ability to do math and eleventh-grade algebra achievement for African American students (Alexander, 2015), nor with ninth-grade algebra achievement among low-achieving students (Kim-Choi, 2015). Higher self-efficacy measured in eleventh grade correlated to higher eleventh-grade algebra achievement for females, especially White females (Howard, 2015; John, 2017); however, no relationship was found between eleventh-grade math self-efficacy and eleventh-grade algebra achievement for Latin Americans, African American females, Latin American females, or Asian American females (Fajardo et al., 2016; Howard, 2015).

In Grade 9, students who perceived math to be useful (math utility) demonstrated lower ninth-grade algebra achievement, especially among high-achieving students, but there was no relationship between low-achieving students' perception of math usefulness and their ninth-grade algebra achievement (Kim-Choi, 2015). Similarly, in eleventh grade, Latin American students' perception of the usefulness of math had no relationship with their eleventh-grade algebra achievement (Fajardo et al., 2016; John, 2017).

We also found that higher math self-efficacy and a higher sense of school belonging were associated with higher ninth-grade algebra achievement (Larrain, 2015). Similarly, higher math interest and consulting a variety of people about mathematics coursework were associated with higher ninth-grade algebra achievement (Carolan & Matthews, 2015). Interestingly, higher math self-efficacy was associated with lower ninth-grade algebra achievement for African American students and was associated with higher ninth-grade algebra achievement for non-African American students (Larrain, 2015). Higher math self-efficacy was associated with higher eleventh-grade algebra achievement for high-achieving Pacific Islander students and lower eleventh-grade algebra achievement for non-Pacific Islander students (Kotok, 2017). However, higher math self-efficacy was associated with lower eleventh-grade algebra achievement for African American students and higher eleventh-grade algebra achievement for non-African American students (Larrain, 2015). Larrain (2015) also found that compared to other racial/ethnic groups, African American students reported the highest levels of math self-efficacy, while also having the lowest achievement. Therefore, one explanation for this finding is that African American students may be overestimating their math self-efficacy and academic skills (Graham, 1994).

In schools where administrators and/or counselors reported that teachers at their school feel responsible that all students learn (collective education goals of a school), students tended to have higher ninth-grade algebra achievement when SES and language variations were accounted for; however, when race/ethnicity variations were controlled for instead, teachers feeling responsible that all students learn no longer related to ninth-grade algebra achievement (Min & Goff, 2016). In addition, as parents' SES increased and teachers felt more responsible that all students learn, ninth-grade algebra achievement trended lower (Min & Goff, 2016).

The parents who reported being involved in school related matters tended to have students with higher eleventh-grade algebra achievement among females, especially Black and White females (Carolan, 2016; Howard, 2015), but there was no relationship between self-reported parent involvement and their eleventh-grade child's algebra achievement for Hispanic or Latin American females, nor Asian American females (Howard, 2015).

Rochmes (2014) found that teachers believing their students can overcome adversity is associated with lower ninth-grade algebra achievement for African American students.

Rochmes also found that students' parent SES changed this relationship, that is, teachers believing that their students can overcome adversity is associated with higher ninth-grade algebra achievement for African American students or multiracial students with higher parent SES.

Overall, we can conclude that students' background factors (i.e., race/ethnicity, SES, gender, or prior achievement) contribute to differences in how the malleable factors at the student, parent, teacher, and school level are related to both ninth- and eleventh-grade algebra achievement. That is, peer engagement, school engagement, math effort, math identity, math interest, math self-efficacy, and math utility have varying effects on algebra achievement based on the students' background factors. Similarly, parent involvement, collective

education goals of a school, and the teacher's belief that students can overcome adversity also have varying effects on algebra achievement based on the students' background factors.

6 Summary

This research contributes to mathematics education methodology by demonstrating how a group of related literatures that uses the same large-scale nationally representative dataset with algebra achievement as the outcome can be examined to explore why results from the same dataset may differ across studies. We found that a student's race/ethnicity, SES, gender, and/or prior achievement plays a role in their algebra achievement and helps to explain why results from the same dataset may differ across studies. Stakeholders can account for these differences in their plans and actions by focusing on those malleable student-, teacher-, school-, and parent-level factors associated with algebra achievement that pertain to the racial/ethnic, SES, gender, or prior achievement make-up of the student populations they serve. Below is a summary of the findings for students, teachers, schools, and parents that were associated with higher algebra achievement.

- Students: gaining learning experiences in advanced mathematics courses earlier in their high school career (especially in algebra). Having explicit plans for mathematics course-taking and college and consulting with a variety of people and/or family members regarding those plans.
- Teachers: supporting all students and emphasizing mathematics concepts over procedures. Having a high expectation for the overall mathematics achievement of their class, and refraining from perceiving students from minority backgrounds as less mathematically inclined or competent.
- Schools: preparing all students to take advanced mathematics courses earlier, noting that some students may need a concurrent support course. Having students take at least four years of math and at least three years of science.
- Parents: maintaining high expectations for their child's academic attainment.

6.1 Recommendations

Given the findings, we suggest that future interventions or professional developments should explore efforts to influence teachers' beliefs away from the damaging perception that students from minority or lower SES backgrounds are less mathematically inclined or competent. These beliefs can have large impacts on minority students' and students with lower SES's opportunities to learn crucial mathematics (i.e., algebra). This belief about students from lower SES backgrounds needs to be addressed at the international level across countries with varying SES inequalities, so as to improve students with lower SES opportunities to learn algebra.

Given the findings that schools who offer advanced placement (AP) courses and/or have higher enrollment in these courses are associated with higher algebra achievement, we can conclude that a school's AP course enrollments may be an index for students taking more mathematics courses. Internationally, schools should focus on providing high school students with opportunities for meaningful learning experiences in mathematics and preparing

all students to have the opportunity to take advanced mathematics courses. This can be accomplished with professional development designed to support committed teachers and their efforts in teaching meaningful mathematics in all mathematics courses for all students regardless of their SES level.

Given the findings, future research may explore the benefits of schools providing incentives to bring in parents as part of their students' coursework and college planning and efforts to maintain high academic expectations. These plans should consider the support needed (e.g., time and resources) for families of varying socioeconomic levels.

Considering ORT principles (4) and (5)—which posit we should focus on incremental progress or reasonable outcomes rather than on comprehensive change and on maximizing progress with available resources while considering student populations (e.g., race, ethnicity, SES; Anderson, 2015)—the above summary items can be reasonably accomplished. We recommend that researchers take these ideas and use them to guide interventions and address public policy concerns regarding improving high schoolers' algebra achievement, ultimately to recruit highly capable individuals to pursue careers in STEM fields.

6.2 Limitation of the study

The limitations for this study are (1) the data were collected within the USA. and are meant to be nationally representative of the US ninth -graders. Hence, the outcome of this study pertains only to the country of origin; (2) the sources of articles were not limited to academic journals, because the authors wanted to alleviate publication bias by including published and unpublished studies in the analysis, that is, the sources included eight journal articles, twelve dissertations, and one book chapter; (3) the data analysis focused on the level of positive/negative associations or relationships of the factors only because the included studies did not provide sufficient information to conduct a comparison of effect sizes; and (4) the data that this systematic review is based on are correlational in nature, and thus, strong caution is suggested when considering implications for practice.

6.3 Methodological contribution

In this study, the authors described how a systematic review can be used to analyze and synthesize a group of related literatures that utilized data from one nationally representative dataset on a specific research question(s) to explore why results from the same dataset may differ across published and unpublished studies.

The systematic review consisted of the following: searching the literature, appraising the studies, and conducting the analysis. The systematic search of the literature should consist of both published and unpublished studies (i.e., journal articles, book chapters, conference proceedings, dissertations) and incorporate the four stages of data collection: identification, screening, eligibility, and inclusion. The included studies should be relevant to the research purpose and they should share high-quality methodology. The quality of each study's methodology should be evaluated by appraising whether the sample was clearly described, whether a clear definition was provided for the variables, whether the dataset items used to create the variables was provided, and whether an appropriate statistical approach was used. Given this appraisal, all the studies can be given equal weighting in the analysis. The analysis should then examine the similarities and differences across the studies (i.e., sample used, dataset's items used, control variables, the variables' significance level).

This study found that any apparent differences across the studies' results could be explained by the different subpopulations (e.g., racial/ethnic, gender, SES, or achievement groups) they examined. This underscores the importance of carefully considering student populations when interpreting research findings and using them to model educational reforms.

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